body, it approaches *Spiochætopterus*. While it conforms to the Spionidæ in the structure of its body-wall and bristles, it differs in regard to the absence of the dorsal branchiæ; and further, the short, pinnate and ciliated anterior branchial organs of *Prionospio* appear to be the nearest approach to its elongated tentacles. In the mechanism of its proboscis and in the structure of its snout and circulatory organs, again, it presents features *sui generis*.

III. "On a new Form of Tangential Equation." By John Casey, LL.D., F.R.S., Professor of Higher Mathematics in the Catholic University of Ireland. Received January 24, 1877.

(Abstract.)

If a variable line make an intercept ν on the axis of x, and an angle ϕ with it on the negative side, the equation of this line will be

$$x+y\cot\phi-\nu=0.$$

The quantities ν and ϕ will determine the position of the line, and may therefore be called its coordinates; hence any relation between ν and ϕ, such as $\nu=f(\phi)$, will be the tangential equation of a curve which is the envelope of the line.

The equation $\nu=f(\phi)$ forms the subject of this paper. It is remarkable for the facility with which it can be transformed into the ordinary Cartesian and tangential equation, as well as into the polar and intrinsic equation of a curve. In a great variety of cases it gives, in a simple form, results which, by other methods, are very cumbersome or nearly impracticable. I have illustrated it throughout by numerous examples, many of which are of historical interest.

The following is an outline of the contents of the paper:—

Chapter I. shows how to transform Cartesian and polar equations into the form $\nu=f(\phi)$. In the course of the investigation a remarkable system of curves of the *nth* class, which are concomitants to any curve of the *nth* degree, are introduced, and their leading properties investigated.

Chapters II., III. are occupied with the transformation of the intrinsic equation, and *vice versâ*, and some allied subjects. In these chapters the whole theory of evolutes, involutes, curvature, &c. are fully considered.

Chapters IV., V. are devoted to the investigation of the properties of cycloids and hypocycloids by their tangential equations. A large number of new properties of these curves are given. The following may be taken as specimens:—1st. If three tangents to a cycloid be given, the envelope of the tangent at its vertex is a parabola. 2nd. If two tangents to a cycloid contain a given angle, the locus of the centre of the circle described about the triangle formed by the two tangents and their chord of contact is a right line.

Chapter VI. contains the theory of positive and negative pedals. The

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following is a remarkable theorem on this part of the subject. The positive pedal of a bicircular quartic is the inverse of its negative pedal if the centre of one of its circles of inversion be taken as origin.

Chapter VII. is the last. It contains the theory of reciprocating curves from their tangential and intrinsic equation. Thus, if $\nu=f(\phi)$ be the tangential equation, its reciprocal is in polar equations

$$\rho=\frac{k^2}{f(\phi)\sin\phi}.$$

Again, if $s=f(\phi)$ be the intrinsic equation of a curve, the polar equation of its reciprocal is

$$\frac{k^2}{\rho}=\left(\frac{d^2}{d\phi^2}+1\right)^{-1}f'(\phi).$$

This chapter contains also the theory of parallel curves. The following is a remarkable property of these curves :—Every focus of any order of the original curve is a focus of the next highest order of the parallel curve.

The last problem discussed in the paper is the rectification of bicircular quartics by elliptic integrals, and the method can be extended to sphero-quartics. This problem, so far as the author is aware, is now solved for the first time.

The paper is enriched by the addition of a very important annex by Professor Cayley.

IV. "Addition on the Bicircular Quartic." By A. Cayley, LL.D., F.R.S., Sadlerian Professor of Mathematics in the University of Cambridge. Received January 24, 1877.

(Abstract.)

Prof. Casey communicated to me the MS. of the foregoing memoir, and he has permitted me to make to it the present addition, containing further developments on the theory of the bicircular quartic.

Starting from his theory of the fourfold generation of the curve, Prof. Casey shows that there exist series of inscribed quadrilaterals, ABCD, whereof the sides AB, BC, CD, DA pass through the centres of the four circles of inversion respectively; or (as it is convenient to express it) the pairs of points (A, B), (B, C), (C, D), (D, A) belong to the four modes of generation respectively, and may be regarded as depending upon certain parameters (his θ, θ', θ'', θ''', or say) ω_1, ω_2, ω_3, ω respectively, any three of these being in fact functions of the fourth. Considering a given quadrilateral ABCD, and giving to it an infinitesimal variation, we have four infinitesimal arcs, AA′, BB′, CC′, DD′; these are in fact differential expressions, AA′ and BB′ of the form $M_1 d\omega_1$, BB′ and CC′ of the form $M_2 d\omega_2$, CC′ and DD′ of the form $M_3 d\omega_3$, DD′ and AA′ of the form $M d\omega$;